Large Force Fluctuations in a Flowing Granular Medium

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We report the characteristics of the temporal fluctuations in the local force delivered to the wall of a 2D hopper by a granular medium flowing through it. The forces are predominantly impulsive at all flow rates for which the flow does not permanently jam. The average impulse delivered to the wall is much larger than the momentum acquired by a single particle under gravity between collisions, reflecting the fact that momentum is transferred to the walls from the bulk of the flow by collisions. At values larger than the average impulse, the probability distribution of impulses is broad and decays exponentially on the scale of the average impulse, just as it does in static granular media. At small impulse values, the probability distribution evolves continuously with flow velocity but does not show a clear signature of the transition from purely collisional flow to intermittently jamming flows. However, the time interval between collisions tends to a power law distribution, $P(\tau) \sim \tau^{-3/2}$, thus showing a clear dynamical signature of the approach to jamming.

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Sand, rather than a liquid, is chosen for the contents of an hourglass because the rate of efflux from a column of sand flowing under gravity does not depend on the height to which it is filled. Likewise, in a static column of sand, the pressure at a given depth is independent of the height of the column above it [1]. In both cases, this is because the weight of the sand in the interior of the column is borne by the lateral containing walls. In the case of a static granular column, experiments [2, 3, 4], theory [5, 6], and numerical simulations [3, 7, 8] have all shown that stresses in the bulk are transmitted to the walls in a very spatially inhomogeneous fashion. This is demonstrated by two independent observations: first, the distribution of forces, $P(f)$, at the boundaries of the medium is exponential, rather than gaussian [9, 10] Secondly, grains that are highly stressed are organized into filamentary, linear structures called 'force chains' that carry a large fraction of the stress [2, 3, 4].

In this article we report experiments directed toward producing a complementary understanding of a flowing granular medium. The force chains in a static medium are unstable to perturbations perpendicular to their length; when jostled by other grains moving in a flow, do they merely become short-lived, or do they melt away entirely? Concomitant to this presumed annealing away of the stress inhomogeneities, does the broad, exponential force distribution in the static case become gaussian when grains move and rearrange? The answers to these questions have broad consequences for the prospects of applying continuum theories to flowing granular matter. If indeed the bounding walls of a medium can communicate with the interior by transient force chains, then any approach to a continuum limit must take into account these long length scales. Previous analyses of the force distributions in a static granular medium [5], show that an exponential at large forces may be obtained from scalar models in which the weight of a bead is borne by the beads beneath it in some random proportion. While force chains do not naturally emerge from these models it is tempting to speculate that force chains and broad force distributions are related manifestations of the large stress inhomogeneities in a granular medium. This point of view is also suggested by recent simulations that show a simultaneous narrowing of force distributions and a blurring of the force chains [8], as static granular media are subjected to compression. On the other hand, a recent proposal [11] for a unified description of jamming in thermal as well as non-thermal systems identifies signatures of the approach to a jammed state in the force distribution, $P(f)$. In contrast to earlier models, they suggest that loss of mobility is due to the formation of force chains, whose presence is most clearly signalled by the scarcity of unstressed regions (i.e. a dip or plateau in $P(f)$ at $f < \bar{f}$, the average force) rather than by the exponential tails at large force values. The formation of a plateau in $P(f)$ has been also been reported in a recent simulation [12] of grains flowing down an inclined chute.

In this article we report measurements of the temporal fluctuations of the forces delivered to the side walls of a hopper (shown in Fig.1a) which contains a 2D flow of steel balls with diameter d=3.125mm. The sides of the hopper are at a fixed angle of 10° to the vertical, so that there are no locations within the cell where the flow stagnates. A transducer (PCB Piezotronics model 209C01) sits flush against one of the sidewalls of the hopper and measures forces normal to that wall. The diameter of the head of the transducer $\approx d$; thus there is only one ball against the transducer at any time. For most of the measurements we report, the transducer is located at a

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FIG. 2: (a) Impulse histograms on a log-linear scale for various V_f (all in cm/s): 9.4(X), 11.7(+), 14.7(●), 17.7(♦), 23.0(▼), 28.4(▲), 33.1(○), 37.6(▽), 44.5(△), 50.1(◊) and 60.0(□). The flow velocities correspond to opening sizes ranging from $a = 3d$ to $a = 16d$. The data at 9.4 cm/s and 23 cm/s correspond to the transducer located higher up in the flow. (b) Impulse histograms of Fig.2a scaled to the average impulse $\bar{I}$ for each flow velocity.

FIG. 3: (a)Probability distributions, $P(\tau)$, of the time intervals τ between collisions, on a log-log scale for a flow velocity of $V_f = 14.7 cm/s$ corresponding to $a = 3.3d$. The solid line corresponds to a power law $P(\tau) \sim \tau^{-3/2}$. The average time interval between impulses $\bar{\tau}$, is marked in the figure. (b) $P(\tau)$ on a log-log scale for different flow velocities, V_f (all in cm/s): 9.4(X), 11.7(+), 14.7($\bullet$), 17.7($\blacklozenge$), 23.0($\blacktriangledown$), 28.4($\blacktriangle$), 33.1($\circ$), 37.6($\bigtriangledown$), 44.5($\bigtriangleup$), 50.1($\Diamond$) and 60.0($\Box$). The corresponding opening sizes range from $a = 3d$ to $a = 16d$. The data at 9.4 cm/s and 23.0 cm/s are taken with the transducer located higher up in the flow. The curves are displaced vertically for clarity. The solid line is a power law: $P(\tau) \sim \tau^{-3/2}$

$\sim 20msecs$, which lies beyond the largest time interval for which we are able to obtain good statistics. However, if the power-law of $\tau^{-3/2}$ is indeed the asymptotic shape of the distribution, then the mean time interval $\bar{\tau} = \int P(\tau)d\tau$ tends to diverge just as in a glass transition (even though the average time computed from a finite, albeit large, data set shows a relatively innocuous dependence on V_f as in Fig. 1c). At even slower flow velocities (the two curves above the solid line in Fig. 3b), there are increasingly frequent instances of sustained contact: at $V_f = 11.7cm/s$ and $14.7cm/s$ the percentage of the time spent in sustained contact is 9% and 4%, respectively, compared to 0.3% at $V_f = 44.5cm/s$. This leads to ambiguities in defining the time interval between collisions possibly resulting in the dashed curves being even broader than the power-law of $\tau^{-3/2}$. A simulation of gravity-driven channel flows [17] also showed a power-law distribution of collision times with an exponent of close to 3. In that simulation, however, they were able to compute the distribution of collision times for particles in the bulk, averaged over time, whereas in the experiment we only have access to the collisions at a fixed location on the wall and cannot keep track of any correlations that are convected down the flow with individual balls.

In summary, our data show that the collisional regime extends very close to the threshold of jamming. The broad, exponential distribution of forces in the jammed, static case extends right through the jamming transition and up to the largest flow velocities that we are able to explore. The small force end of the force distribution does evolve as the flow is slowed down, but the indication of an imminent jammed state appears via a dynamical quantity, namely, the distribution of collision times, $P(\tau)$. The existence of transient force chains in a flowing medium remains unresolved by the measurements reported here, though it is clear that if force chains are intimately connected with exponential force fluctuations, then they must persist in even the most rapid flows we have investigated. We are pursuing this issue further by direct measurement of spatial correlations between different points in the flow.

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[1] H. M. Jaeger, S. R. Nagel, R. P. Behringer, Rev. Mod. Phys. **68**, 1259 (1996).
[2] P. Dantu, Ann. Pont Chaussees IV, 144 (1967); T. Travers et al., Europhys. Lett. **4**, 329 (1987).
[3] C. H. Liu et al., Science **269**, 513 (1995).
[4] D. Howell, R. P. Behringer, C. Veje, Phys. Rev. Lett. **82**, 5241 (1999).
[5] S. N. Coppersmith et al., Phys. Rev. E **53**, 4673 (1996).
[6] M. E. Cates, J. P. Wittmer, J. P. Bouchaud, P. Claudin, Chaos **9**, 511 (1999).
[7] F. Radjai, M. Jean, J. J. Moreau, S. Roux, Phys. Rev. Lett. **77**, 274 (1996).
[8] H. A. Makse, D. L. Johnson, L. M. Schwartz, Phys. Rev. Lett. **84**, 4160 (2000).
[9] D. M. Mueth, H. M. Jaeger, S. R. Nagel, Phys. Rev. E **57**, 3164 (1998).
[10] B. Miller, C. OHern, R. P. Behringer, Phys. Rev. Lett. **77**, 3110 (1996).
[11] C. S. OHern, S. A. Langer, A. J. Liu, S. R. Nagel, Phys. Rev. Lett. **86**, 111 (2001).
[12] L.E. Silbert et al., cond-mat/0109124
[13] G. W. Baxter et al., Phys. Rev. Lett. **62**, 2825 (1989).
[14] C. T. Veje , P. Dimon, Phys. Rev. E **56**, 4376 (1997); Phys. Rev. E **54**, 4329 (1996).
[15] G. W. Baxter, R. Leone, R. P. Behringer, Europhys. Lett. **21**, 569 (1993); G. W. Baxter et al., Eur. J. of Mech. B **10**, 181 (1991)
[16] D. L. Blair et al., Phys. Rev. E **63**, 041304-1 (2001).
[17] C. Denniston , H. Li, Phys. Rev. E **59**, R3289 (1999).
[18] The statistics of arch formation at the jamming threshold is discussed in K.To , P.Y. Lai, H.K. Pak, Phys. Rev. Lett. **86**, 71 (2001).